# Acceleration

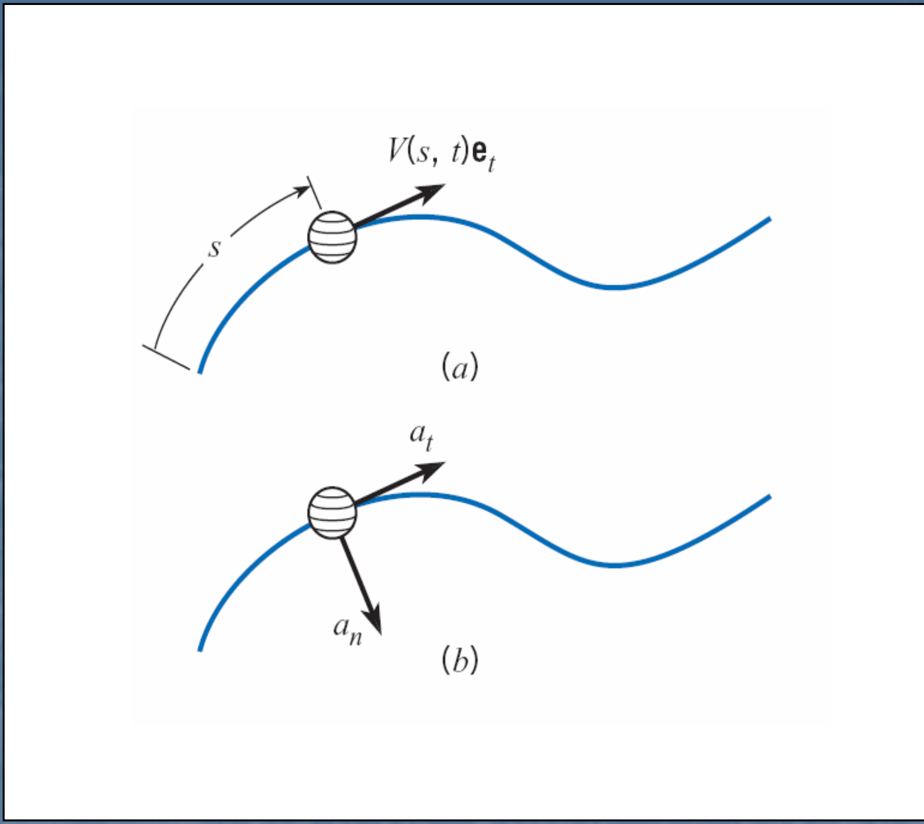
**Lagrangian Approach where**

**the velocity is a function of time only.**

**The velocity of fluid particle**

**can be expressed as**

$$V = V(s,t)e_t$$




**Fluid particle moving on a pathline**

**Where:** $V(s,t)$ is the speed of particle.

$e_t$ is the unit vector of velocity direction.

# Fluid Motion

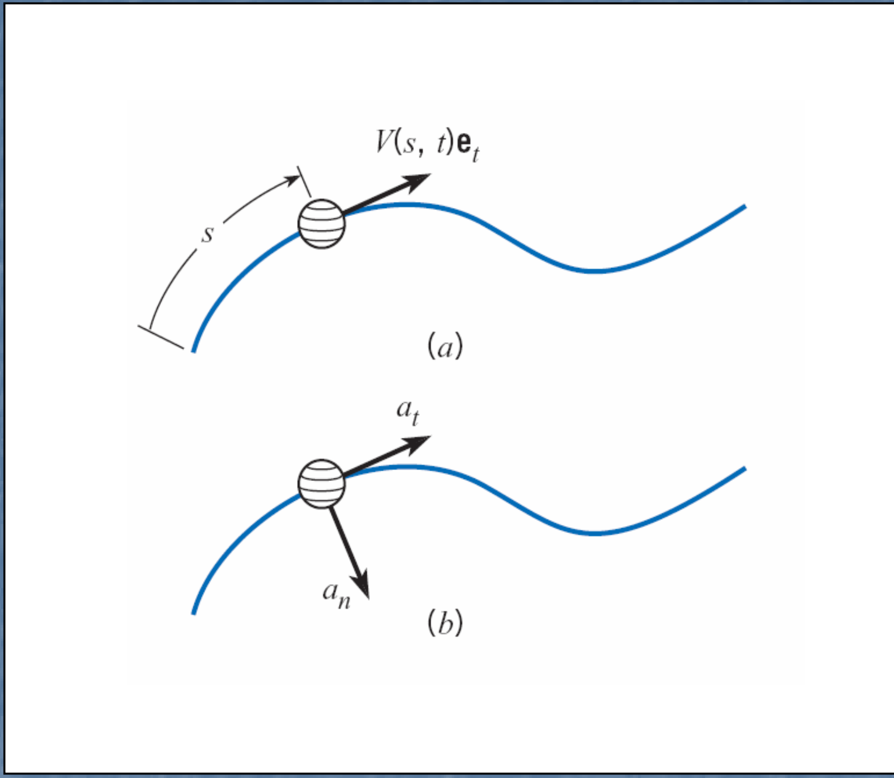
**The velocity of fluid particle can be expressed as**

$$a = \frac{dV}{dt} = \left(\frac{dV}{dt}\right)e_t + V\left(\frac{de_t}{dt}\right)$$

$$\left(\frac{dV}{dt}\right)e_t = \frac{dV(s,t)}{dt} = \left(\frac{\partial V}{\partial s}\right)\left(\frac{\partial s}{\partial t}\right) + \left(\frac{\partial V}{\partial t}\right)$$

$$\frac{dV}{dt} = V\left(\frac{\partial V}{\partial s}\right) + \left(\frac{\partial V}{\partial t}\right)$$

$$\left(\frac{de_t}{dt}\right) = \left(\frac{V}{r}\right)e_n$$




**Where:**

$r$ **= radius of local curvature**

$e_n$ **= unit vector that is perpendicular to the pathline**

# Fluid Motion

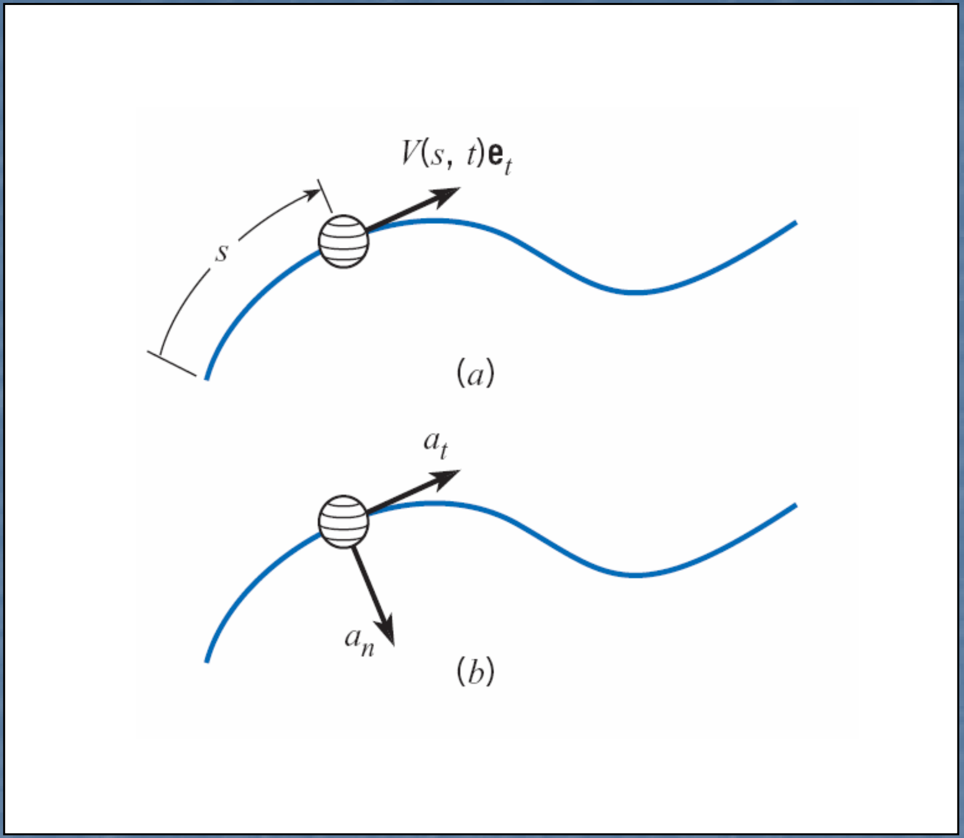
$$a = \left( V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right) e_t + \left( \frac{V^2}{r} \right) e_n$$

$$a = a_t + a_n$$

**Where:**

$$a_t = \left( V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

$$a_n = \left( \frac{V^2}{r} \right)$$



# Fluid Motion

## Acceleration

## Eulerian Approach where the velocity is a function of time and Space.

**Considering the Cartesian coordinates**

$V = ui + vj + wk$ **Where** $u = f(x, y, z, t), \quad v = f(x, y, z, t), \quad w = f(x, y, z, t)$

**The acceleration in the X-direction is given by,**

$$a_x = \frac{du}{dt} = \left(\frac{\partial u}{\partial x}\frac{dx}{dt}\right) + \left(\frac{\partial u}{\partial y}\frac{dy}{\partial t}\right) + \left(\frac{\partial u}{\partial z}\frac{dz}{\partial t}\right) + \left(\frac{\partial u}{\partial t}\right)$$

**The acceleration in the Y-direction is given by,**

$$a_y = \frac{dy}{dt} = \left(\frac{\partial v}{\partial x}\frac{dx}{dt}\right) + \left(\frac{\partial v}{\partial y}\frac{dy}{\partial t}\right) + \left(\frac{\partial v}{\partial z}\frac{dz}{\partial t}\right) + \left(\frac{\partial v}{\partial t}\right)$$

# Fluid Motion

**The acceleration in the Z-direction is given by,**

$$a_z = \frac{dw}{dt} = \left(\frac{\partial w}{\partial x}\frac{dx}{dt}\right) + \left(\frac{\partial w}{\partial y}\frac{dy}{\partial t}\right) + \left(\frac{\partial w}{\partial z}\frac{dz}{\partial t}\right) + \left(\frac{\partial w}{\partial t}\right)$$

**As** $u = \left(\frac{dx}{dt}\right)$ $v = \left(\frac{dy}{dt}\right)$ $w = \left(\frac{dz}{dt}\right)$

$$a_x = \frac{du}{dt} = \left(u\frac{\partial u}{\partial x}\right) + \left(v\frac{\partial u}{\partial y}\right) + \left(w\frac{\partial u}{\partial z}\right) + \left(\frac{\partial u}{\partial t}\right)$$

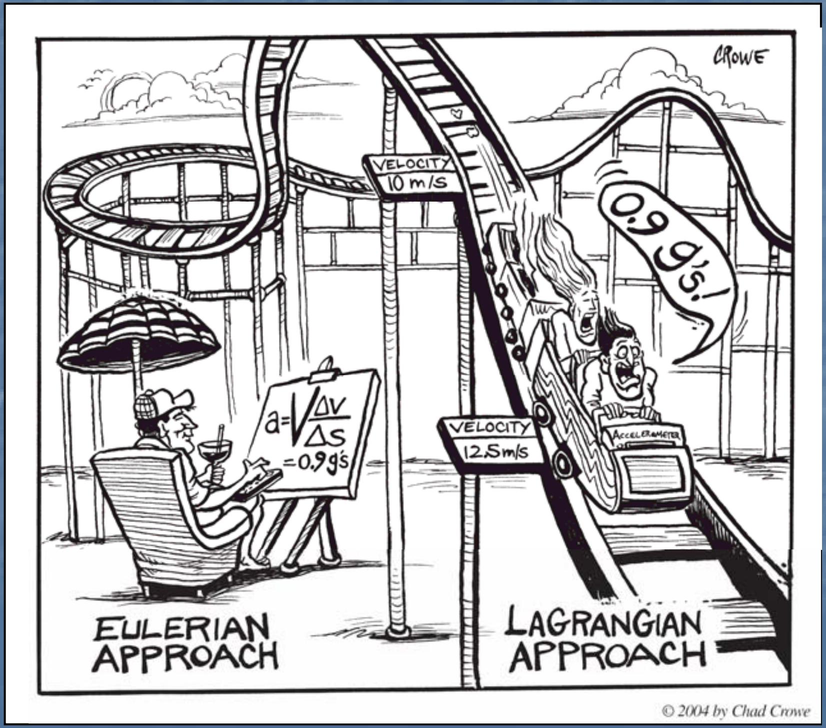
$$a_y = \frac{dy}{dt} = \left(u\frac{\partial v}{\partial x}\right) + \left(v\frac{\partial v}{\partial y}\right) + \left(w\frac{\partial v}{\partial z}\right) + \left(\frac{\partial v}{\partial t}\right)$$

$$a_z = \frac{du}{dt} = \left(u\frac{\partial w}{\partial x}\right) + \left(v\frac{\partial w}{\partial y}\right) + \left(w\frac{\partial w}{\partial z}\right) + \left(\frac{\partial w}{\partial t}\right)$$

# Fluid Motion

**Local Acceleration =** $\left(\frac{\partial u}{\partial t}\right), \left(\frac{\partial v}{\partial t}\right)$ and $\left(\frac{\partial w}{\partial t}\right)$

**Convective Accelerations =** $\left(u\frac{\partial u}{\partial x}\right)$ $\left(v\frac{\partial v}{\partial y}\right)$ $\left(w\frac{\partial w}{\partial z}\right)$



# Fluid Motion

$$V = 2x^2ti + 3xy^2 j + 2xzk$$

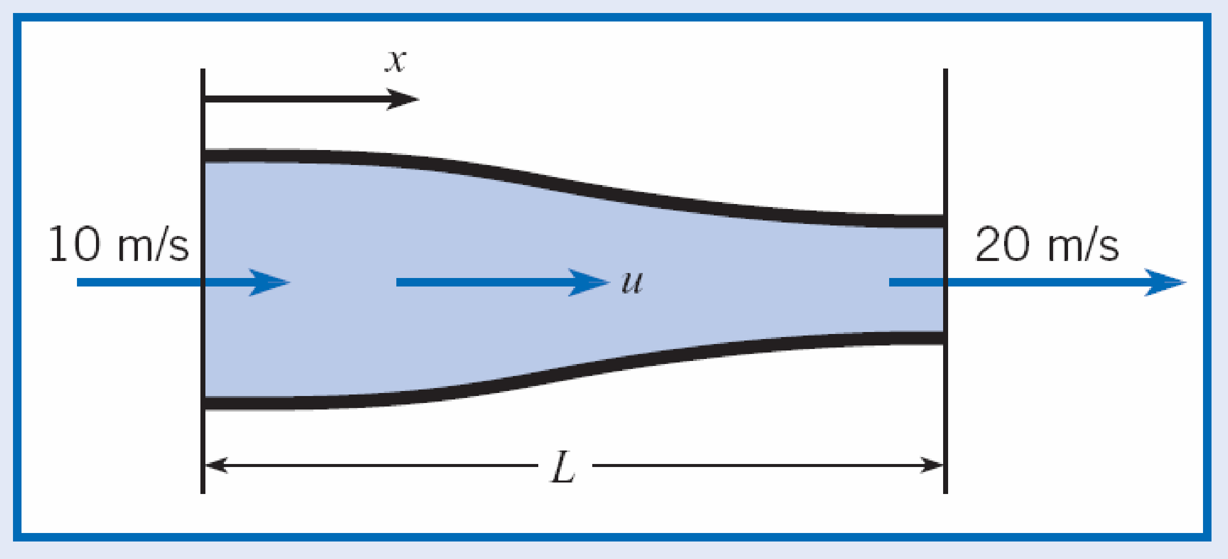
**At point (1,1,1),** $Find\ a_x$

$$a_x = \frac{du}{dt} = \left(u\frac{\partial u}{\partial x}\right) + \left(v\frac{\partial u}{\partial y}\right) + \left(w\frac{\partial u}{\partial z}\right) + \left(\frac{\partial u}{\partial t}\right)$$

# Fluid Motion

## Solved Example

$$u = \frac{u_0}{1 - \frac{0.5x}{L}}$$




$$a_x = \frac{du}{dt} = \left(u\frac{\partial u}{\partial x}\right) + \left(v\frac{\partial u}{\partial y}\right) + \left(w\frac{\partial u}{\partial z}\right) + \left(\frac{\partial u}{\partial t}\right)$$

$$u\frac{\partial u}{\partial x}$$

# END OF LECTURE (3)